



# Programming Languages 2

Homework 7 – WS 18

Tübingen, 6. Dezember 2018

In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

**Handin** Please submit this homework until Thursday, December 13, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

**Groups** You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

**Points** For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:  
1 point, if your submission shows that you tried to solve the task.  
2 points, if your submission is mostly correct.

## Task 1: Records and Variants

For which of the following terms  $t$  does a context  $\Gamma$  and a type  $T$  exist, such that they are well typed. In other words  $\Gamma \vdash t : T$ ? If they exist, please write down  $\Gamma$  and  $T$ . If not, a short note is enough.

1.  $\text{iszero}(s.\text{age})$
2.  $\text{case } s \text{ of } \langle \text{visited} = x \rangle \Rightarrow \text{iszero } x$
3.  $\text{let } s = \langle \text{visited} = \text{false} \rangle \text{ as } \langle \text{visited} : \text{Bool} \rangle \text{ in } \text{iszero } s$

## Task 2: Reduction of Sums and Variants

Reduce the following terms until they reach a normal form. Use the reduction relation from the lecture.

1.  $\text{case}(\text{inl } 5 \text{ as } \text{Nat} + \text{Unit}) \text{ of } \text{inl } x \Rightarrow \text{iszero } x \mid \text{inr } y \Rightarrow y$
2.  $\text{inl}(\text{if true then false else true}) \text{ as } \text{Bool}$
3.  $\text{case}(\langle \text{circle} = 5 \rangle \text{ as } \langle \text{circle} : \text{Nat}, \text{square} : \text{Nat} \rangle) \text{ of } \langle \text{square} = x \rangle \Rightarrow \text{iszero } x$

You do not have to draw a derivation tree for the reduction relation, but you do have to write down all reduction steps.

### Task 3: Explicit Fixed Point

Write a term of type  $(\text{Nat} \times \text{Nat}) \rightarrow \text{Nat}$  that computes the greatest common divisor of two numbers. Use the explicit fixed point operator `fix` or `letrec` as well as any language features presented in the lecture. Assume macro definitions for arithmetic operations and comparison operations on natural numbers (addition, subtraction, division, multiplication, larger than, equals, smaller than, ...).