EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Fachbereich Informatik

Programmiersprachen und Softwaretechnik

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Responsible for the lab
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Programmiersprachen II

Homework 2 – WS 18 Tübingen, 25. Oktober 2018

In order to be admitted to the exam, you have to successfully submit your homework every week, except for 2 weeks. A successful submission is one where you get at least 1 point.

Handin Please submit this homework until Thursday, November 08, either via email to Philipp Schuster (philipp.schuster@uni-tuebingen.de) before 12:00, or on paper at the beginning of the lab.

Groups You can work in groups of up to 2 people. Please include the names and Matrikelnummern of all group members in your submission.

Points For each of the Tasks you get between 0 and 2 points for a total of 6 points. You get:
1 point, if your submission shows that you tried to solve the task.
2 points, if your submission is mostly correct.

Task 1: Derivation trees

We define an example language by the following grammar:

$\langle \textit{term} \rangle$ ::= ‘`zero`’ | ‘`succ`’ $\langle \textit{term} \rangle$ | ‘`false`’ | ‘`true`’
| ‘`iszero`’ $\langle \textit{term} \rangle$ | ‘`if`’ $\langle \textit{term} \rangle$ ‘`then`’ $\langle \textit{term} \rangle$ ‘`else`’ $\langle \textit{term} \rangle$

We define an operational semantics for the language by defining the reduction relation $\longrightarrow$ as the smallest relation $t \longrightarrow t'$, closed under the following derivation rules:

E-Succ
$$\frac{t_1 \longrightarrow t_1'}{\mathrm{succ}\, t_1 \longrightarrow \mathrm{succ}\, t_1'}$$

E-IsZeroZero
$$\mathrm{iszero\ zero} \longrightarrow \mathrm{true}$$

E-IsZeroSucc
$$\mathrm{iszero}(\mathrm{succ}\, t) \longrightarrow \mathrm{false}$$

E-IsZero
$$\frac{t_1 \longrightarrow t_1'}{\mathrm{iszero}\, t_1 \longrightarrow \mathrm{iszero}\, t_1'}$$

E-IfTrue
$$\mathrm{if\ true\ then}\ t_2\ \mathrm{else}\ t_3 \longrightarrow t_2$$

E-IfFalse
$$\mathrm{if\ false\ then}\ t_2\ \mathrm{else}\ t_3 \longrightarrow t_3$$

E-If
$$\frac{t_1 \longrightarrow t_1'}{\mathrm{if}\ t_1\ \mathrm{then}\ t_2\ \mathrm{else}\ t_3 \longrightarrow \mathrm{if}\ t_1'\ \mathrm{then}\ t_2\ \mathrm{else}\ t_3}$$

Which of the rules are computation rules, which of the rules are congruence rules?

Prove that the term $\mathrm{succ}(\mathrm{succ}(\mathrm{iszero}(\mathrm{succ}\,\mathrm{zero})))$ is not in normal form, by giving a derivation tree with root:

$$\frac{?}{\mathrm{succ}(\mathrm{succ}(\mathrm{iszero}(\mathrm{succ}\,\mathrm{zero}))) \longrightarrow \mathrm{succ}(\mathrm{succ}\,\mathrm{false})}$$

Task 2: Deterministic reduction

The language and its reduction relation from Task 1 is non-deterministic which means that there is a term t that reduces in one step to two different terms. Show this, by finding t, t_1 and t_2 such that $t \longrightarrow t1$ as well as $t \longrightarrow t2$. No proof required.
Describe in two sentences an approach for making the reduction relation deterministic.

Task 3: Induction on derivation trees

Let the function size for the language from Task 1 be defined as:

$\mathrm{size}(\mathrm{zero}) = 1$
$\mathrm{size}(\mathrm{succ}\,t_1) = \mathrm{size}(t_1) + 1$
$\mathrm{size}(\mathrm{false}) = 1$
$\mathrm{size}(\mathrm{true}) = 1$
$\mathrm{size}(\mathrm{iszero}\,t_1) = \mathrm{size}(t_1) + 1$
$\mathrm{size}(\mathrm{if}\,t_1\,\mathrm{then}\,t_2\,\mathrm{else}\,t_3) = \mathrm{size}(t_1) + \mathrm{size}(t_2) + \mathrm{size}(t_3) + 1$

Show by induction on the possible derivation trees that from $t \longrightarrow t'$ it follows that $\mathrm{size}(t') < \mathrm{size}(t)$.